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On the Roots of a Determinantal Equation.

By W. H. Metzler.

1. In the American Journal of Mathematics* Dr. Thomas Muir made use of Sylvester's proof of the reality of the roots of Lagrange's determinantal equation to prove the theorem

The n^{thic} equation,

$$\begin{vmatrix} 11-x & 12 & 13 & \dots \\ 21 & 22-x & 23 & \dots \\ 31 & 32 & 33-x & \dots \\ \dots & \dots & \dots & \dots \end{vmatrix} = 0$$

will have all its roots real if in the case of every pair μ, ν *of the* $n-1$ *indices* $2, 3, 4, \dots, n$, *we have*

$$1\mu \cdot \mu\nu \cdot \nu 1 = \mu 1 \cdot \nu\mu \cdot 1\nu$$

and
$$1\mu \cdot \mu 1 = + .$$

There is another theorem quite similar to this which may be proven in a precisely similar manner.

It is as follows:

The n^{thic} equation

$$\begin{vmatrix} 11-x & 12 & 13 & \dots \\ 21 & 22-x & 23 & \dots \\ 31 & 32 & 33-x & \dots \\ \dots & \dots & \dots & \dots \end{vmatrix} = 0$$

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* Vol. XIX, No. 4, pp. 312-318.

will have all its roots pure imaginary[*] *if in the case of every pair* μ, ν *of the* $n-1$ *indices* 2, 3, 4, ... , n, *we have*

$$1\mu\,.\,\mu\nu\,.\,\nu 1 = -\mu 1\,.\,\nu\mu\,.\,1\nu\,,$$
$$1\mu\,.\,\mu 1 = -\,,$$

and $$\lambda\lambda = 0\,. \qquad (\lambda = 1, 2, 3, \ldots\ldots, n)$$

2. In art. 5 of his paper Dr. Muir shows that, in any determinant in which the specified conditions mentioned in the first theorem are true, every term is equal to its conjugate. In exactly the same way it may be shown that, in any determinant in which the specified conditions mentioned in the second theorem are true, every term is equal to its conjugate if the order of the determinant is even and to the negative of its conjugate if the order of the determinant is odd. Consequently, in case the order of the determinant is odd, conjugate terms cancel each other. Again, when the order is odd, self-conjugate terms contain at least one constituent from the principal diagonal and therefore vanish.

It follows that every determinant of odd order, in which the specified conditions of the second theorem are true, vanishes.

3. If now, following Dr. Muir, we consider the equation of the fourth degree, the proof of the second theorem proceeds precisely the same as that given of the first up to the point where the terms of the quartic in x^2 are shown to be alternately positive and negative.

Examining these terms under the conditions of the second theorem, it will be seen, on making use of the results of art. 2, that the coefficient of $-x^6$ is negative, the coefficient of x^4 is positive, the coefficient of $-x^2$ is negative, and the independent term is positive. The quartic, therefore, has all its terms positive, and consequently can have no real positive root. It can have no imaginary roots, for the specified conditions of the first theorem are true in the determinantal equation $f(x)f(-x) = 0$. Hence the roots of the quartic in x^2 are all negative, and therefore the quartic in x has all its roots pure imaginary.

SYRACUSE UNIVERSITY, *March*, 1898.

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[*] If n is odd and $= 2\rho + 1$, one root will be zero and the other 2ρ will be pure imaginary.